## Covariance and Correlation

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3.0 What We Need to Know When We Finish This Chapter

This chapter develops simple ways to measure the direction of the association and the reliability of the association between two variables in a sample. Here are the essentials.

1. Equation (3.7), section 3.2: The sample covariance is

$$
\operatorname{COv}(X, Y)=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{n-1} .
$$

It is symmetric with regard to $X$ and $Y$.
2. Exercise 3.2: The sample covariance is not invariant to scale:
$\operatorname{COV}(a X, b Y)=a b \operatorname{COV}(X, Y)$.
3. Section 3.3: A derivation begins with an accepted definition and concludes with an implication that is usually not obvious and is often very useful.
4. Equation (3.13), section 3.4: The sample correlation coefficient is $\operatorname{CORR}(X, Y)=\frac{\operatorname{COV}(X, Y)}{\operatorname{SD}(X) \operatorname{SD}(Y)}$.

It is symmetric with regard to $X$ and $Y$.
5. Exercise 3.4: The sample correlation is invariant to scale.

$$
\operatorname{CORR}(a X, b Y)=\operatorname{CORR}(X, Y) .
$$

